

# Effect of Constraints on Optimum Approach and Departure Paths for VTOL Terminal Operations

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A VTOL aircraft is modeled as a point mass moving in a vertical plane acted upon by thrust, gravity, lift, and drag. Approach and departure paths are studied under various constraints, using a hybrid computer simulation. The pilot has control over the angle of attack, thrust magnitude, and thrust direction. Fuel consumption is taken to be proportional to the time integral of the thrust. Constraints are placed on the approach path plus the maximum value of velocity, acceleration, and angle of attack. Data are presented to show how the fuel cost varies as a function of the constraints imposed. The most significant consideration for fuel economy is minimization of the time during which the aircraft flies below conventional stall speed. In general, the steeper the approach or departure path, the greater the fuel cost. Fuel-optimum approaches call for high descent rates at low altitude. When the rate of descent is constrained, the fuel increases but loses its sensitivity to approach angle. Automatic velocity control is necessary to maintain the glide path during steep approaches.

## Nomenclature

$AR$	= aspect ratio
$C_{D0}$	= profile drag coefficient
$C_{DI}$	= induced drag coefficient
$C_{L\alpha}$	= lift curve slope
$D$	= drag, lb
$e$	= wing efficiency factor
$g$	= gravitational acceleration, ft/sec <sup>2</sup>
$h$	= vertical coordinate or altitude, ft
$i$	= thrust direction relative to flight reference line
$L$	= lift, lb
$m$	= aircraft mass, sec <sup>2</sup> /lb-ft
$\dot{m}_a$	= air mass flow rate, sec/lb-ft
$S$	= wing reference area, ft <sup>2</sup>
$T$	= thrust, lb
$t$	= time, sec
$t_f$	= flight time, sec
$V$	= velocity, fps
$V_e$	= engine exhaust velocity, fps
$V_{eq}$	= equilibrium velocity, fps
$W$	= aircraft weight, lb
$x$	= horizontal coordinate or range, ft
$\alpha$	= angle of attack
$\gamma$	= flight path angle
$\gamma_{eq}$	= equilibrium flight path angle
$\delta$	= thrust direction relative to horizontal
$\theta$	= pitch angle
$\kappa$	= ratio of cruise miles to lb-sec of thrust, mile/lb-sec
$\rho$	= air density, sec <sup>2</sup> /lb-ft <sup>4</sup>

## Introduction

THE objective of this work has been to establish optimum approach and departure paths for VTOL terminal operations. The VTOL vehicle was modeled as a point mass moving in a vertical plane acted upon by thrust, gravity, lift, and drag. The pilot has control over angle of attack, thrust magnitude, and thrust direction. Fuel consumption was taken to be proportional to the time integral of the thrust. Two previous studies<sup>1,2</sup> using the same analytical

model have shown that the unconstrained optimum calls for the aircraft to dive underground during both the initial acceleration after takeoff and the final deceleration prior to landing. In this work the aircraft has been constrained not to descend below the ground. The angle of attack has been constrained not to exceed a critical value for stall. The maximum acceleration has been limited for structural safety and modest passenger comfort. The effect of constraining the angle of approach and departure has also been investigated.

The particular VTOL aircraft simulated is designed<sup>3</sup> to transport 80 passengers for a range of 200 miles at a cruise altitude of 20,000 ft. This particular vehicle was chosen in order to be able to compare the simulator results with those obtained by Mehra and Bryson.<sup>1</sup> Agreement was taken as a test of the validity of the simulation. Constrained optima were constructed by trial and error on the simulator guided by the conclusions drawn from steady-state models. The results have not been shown to satisfy any mathematical criteria of optimality.

## Hybrid Computer Simulation

The diagram in Fig. 1 shows how the various quantities are defined. The angles are positive in the direction of the arrows. The equations of motion resolved in the  $x-h$  coordinate frame are

$$m\ddot{x} = T \cos \delta - D \cos \gamma - L \sin \gamma + \dot{m}_a V [\cos \delta - \cos \gamma] \quad (1)$$

$$m\ddot{h} = T \sin \delta - D \sin \gamma + L \cos \gamma - mg + \dot{m}_a V [\sin \delta - \sin \gamma] \quad (2)$$

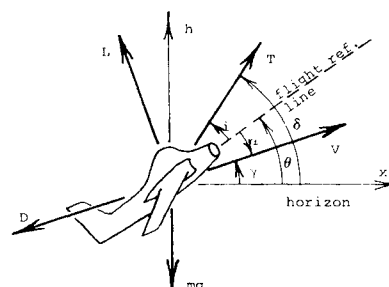


Fig. 1 Force diagram for aircraft.

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where

$$L = 1/2 \rho S C_{L\alpha} \alpha V^2 \quad (3)$$

$$D = 1/2 \rho S V^2 (C_{D0} + C_{DI} \alpha^2) \quad (4)$$

$$V = (\dot{x}^2 + \dot{h}^2)^{1/2} \quad (5)$$

$$\gamma = \tan^{-1}(\dot{h}/\dot{x}) \quad (6)$$

$$\alpha = \theta - \gamma \quad (7)$$

$$\delta = i + \theta \quad (8)$$

$$C_{DI} = C_{L\alpha}^2 / \pi e A R \quad (9)$$

$$\dot{m}_a = T / [(65) (32.2)] \text{ sec/lb-ft} \quad (10)$$

if  $T$  is in lb. The  $\dot{m}_a V$  terms result from the directional change in the momentum of the air as it passes through the engine. These same equations, resolved in velocity-flight-path-angle coordinates, were used in Ref. 1. The advantage of the equations in the form given is that there is no singularity at zero velocity.

The aircraft mass is taken to be constant during the takeoff and landing. The air density varies with altitude according to

$$\rho = \rho_{SL} (1 - 0.6875 \times 10^{-5} h)^{4.2561} \quad (11)$$

where  $h$  is in feet.

The optimum takeoff and landing is the one that minimizes fuel consumption. Since the fuel mass flow rate is generally proportional to the thrust, the quantity to be minimized is the integral

$$\int_0^{t_f} T dt$$

The takeoff and landing consist of the complete ascent and descent, respectively, between the ground and cruise altitude.

It is assumed that there is closed loop control over the pitch angle so that making either  $\theta$  or  $\alpha$  the control variable is equivalent. The other controls are thrust magnitude and thrust direction. The wings are rigidly attached to the aircraft, but the engine thrust direction can be rotated relative to the wings through the full  $360^\circ$ . It is assumed that minimum thrust is zero, the maximum is 25% greater than the aircraft weight. The cases for maximum thrust 10 and 5% greater than the aircraft weight were also considered. The thrust varies with altitude according to

$$T = T_{SL} (1 - 0.55h/30,000) \quad (12)$$

where  $T_{SL}$  is the sea-level thrust, and  $h$  is in feet.

Table 1 gives the value for the aircraft parameters that were used (Ref. 1). The angle of attack that provides the maximum lift to drag ratio is found as follows:

$$L/D = C_{L\alpha} \alpha / (C_{D0} + C_{DI} \alpha^2) \quad (13)$$

$$\frac{d(L/D)}{d\alpha} = \frac{C_{L\alpha}}{C_{D0} + C_{DI} \alpha^2} - \frac{2C_{L\alpha} C_{DI} \alpha^2}{(C_{D0} + C_{DI} \alpha^2)^2} = 0 \quad (14)$$

$$\alpha_{(L/D)\max} = (C_{D0}/C_{DI})^{1/2} = 6.8^\circ$$

The maximum angle of attack is taken to be  $15^\circ$ , which is

Table 1 Aircraft parameters

Parameter	Value
$C_{D0}$	0.027
$C_{L\alpha}$	5.73
$e$	0.9
$AR$	6.0
$S$	421 ft <sup>2</sup>
$W$	56,900 lb
$m$	1765 sec <sup>2</sup> /lb-ft
$C_{DI}$	1.93

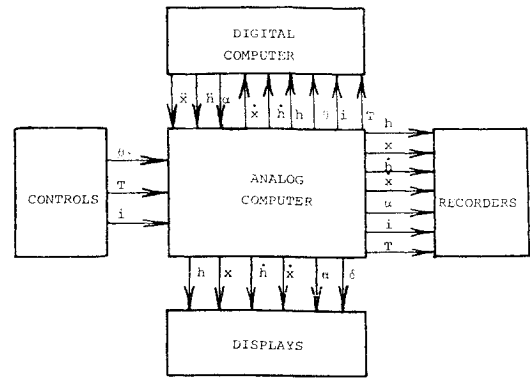


Fig. 2 Simulator flow diagram.

slightly more than twice  $\alpha_{(L/D)\max}$ .

The velocities for stall and maximum lift to drag ratio are found by assuming the aircraft is in unaccelerated level flight with zero thrust angle. Thus, equating lift with weight gives

$$V_{(L/D)\max} = [2W/(\rho S C_{L\alpha} \alpha_{(L/D)\max})]^{1/2} \quad (16)$$

$$V_{\text{stall}} = [2W/(\rho S C_{L\alpha} \alpha_{\max})]^{1/2} \quad (17)$$

Consider (17) to be the definition of conventional stall speed. The sea-level stall speed is 280 fps. The sea-level velocity for maximum lift to drag ratio is 420 fps. It was assumed that the aircraft cruises at the speed for maximum lift-to-drag ratio which is the speed for maximum endurance. The thrust required for cruise  $T_{CR}$  is found from Eq. (1) with  $\dot{x} = \gamma = i = 0$ . This gives  $T_{CR} = 4580$  lb. The number of cruise miles that can be traveled per lb-sec of thrust  $\kappa$  is given by

$$\kappa = V_{(L/D)\max} / T_{CR} = 2.32 \times 10^{-5} \text{ mile/lb-sec} \quad (18)$$

Thus, each second of thrusting at the maximum thrust of  $1.25W$  is equivalent to 1.65 miles of range. Thrusting at one weight for 1 min is equivalent to 79 cruise miles. A larger value for  $\kappa$  results if the velocity for maximum range is used instead of the velocity for maximum endurance.

Figure 2 shows a flow diagram of the system used to simulate the VTOL aircraft. The analog computer used was a GPS 290T. It received values for  $\dot{x}$  and  $\dot{h}$  from the digital computer and integrated them to give  $x$ ,  $h$ ,  $x$ , and  $h$ . The analog computer received signals from the controls to provide values for  $\theta$ ,  $T$ , and  $i$ . It integrated the thrust to give the fuel consumption and also provided the necessary information for the displays and recorders. The digital computer used a PDP-8. It received values for  $x$ ,  $h$ ,  $T$ ,  $i$ , and  $\theta$  from the analog computer. It then computed the right-hand side of the equations of motion [Eqs. (1) and (2)], which yielded values for  $\dot{x}$  and  $\dot{h}$ . The cycle was completed every 3.9 msec.

The three controls for the VTOL aircraft came from a three-degree-of-freedom control stick. The diagram in Fig. 3 illustrates how the controls were actuated by the stick. The quantities that were displayed to aid in controlling the aircraft were  $\dot{h}$ ,  $\dot{x}$ ,  $h$ ,  $x$ ,  $\alpha$ , and  $\delta$ . The altitude and range were shown by an  $x$ - $y$  plotter. The other variables were displayed by a Brush recorder. Additional details of the simulation are available in Ref. 4.

### Steady-State Thrust Requirement

Neglecting the small forces due to the directional change in the momentum of the air passing through the engine, the equations of motion for the vehicle give the thrust in vector form as

$$T = \dot{V} - L - D - W \quad (19)$$

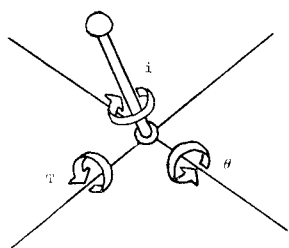


Fig. 3 Control stick.

The fuel cost is the time integral of the magnitude of the thrust.

$$|T| \leq |\dot{V}| + |L| + |D| + |W| \quad (20)$$

The maximum contribution to the cost due to each term in (20) can be estimated in cruise miles. If  $\dot{V}$  is in a fixed direction and  $V$  changes monotonically, then the cost of the acceleration term will be less than the total velocity change expressed in cruise miles. The cost due to the acceleration term alone in going from rest to stall speed is less than 10 cruise miles. The cost of the lift term is negligible since thrust will almost never be used to counter lift. The lift is used to counter the weight, and thrust is used to counter the drag. The maximum value of the drag (neglecting the

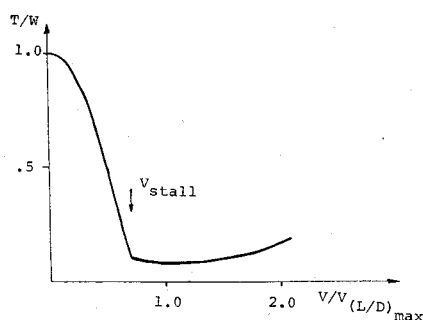


Fig. 4 Thrust required for level flight.

intentional use of high drag devices wherein the drag is not cancelled by thrust) is at stall speed where the cost is about 9 cruise miles each min. At velocities below stall speed, thrust must be used to counter both weight and drag. The cost of countering the weight is 79 cruise miles per min and is clearly the most significant cost. The comparatively low integrated cost of the acceleration term seems to justify the use of steady-state models that neglect acceleration and equate the thrust to the sum of lift, drag, and weight. These models are classical to aircraft performance theory and can be found in any standard text such as Ref. 5. By using such models, the thrust required for level flight can be expressed as a function of velocity as shown in Fig. 4. The graph has been extended to velocities below conventional stall speed by assuming that the angle of attack is held at its maximum value and the thrust direction adjusted to achieve equilibrium. A glance at Fig. 4 shows that the most important consideration for fuel economy is the minimization of the

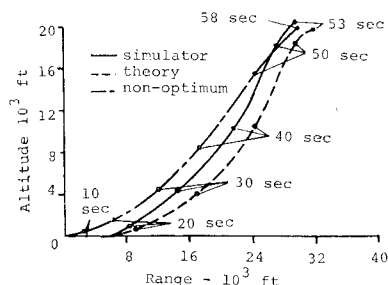


Fig. 5 Flight path for unconstrained takeoff.

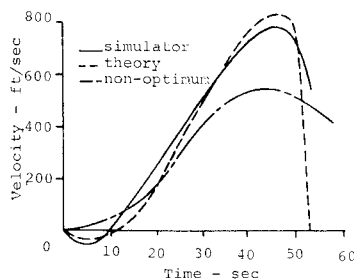


Fig. 6 Vertical velocity for unconstrained takeoff.

time spent at velocities below conventional stall speed. If considerable time must be spent in this speed range, then it is important to have the angle of attack at its maximum value because it minimizes the thrust required at these speeds under steady-state conditions. The same conclusion has been reached by Gallant et al.<sup>6</sup> for the tilting VTOL aircraft; they suggested independent control over wing position and thrust direction in order to maintain the angle of attack near maximum. With angle of attack fixed, the specification of the flight path uniquely determines the remaining two controls (thrust magnitude and thrust direction).

### Optimum Takeoff

To duplicate Mehra and Bryson's results, the aircraft was controlled manually to match the time histories for  $\alpha$  and  $i$  as closely as possible. Mehra and Bryson used Eqs. (1) and (2) resolved in velocity and flight-path-angle coordinates. To avoid the singularity at  $V = 0$ , a small initial velocity of 50 fps with an initial flight path angle of  $7^\circ$  had to be assumed. These initial conditions were used for the duplication. The time to climb to 20,000 ft was minimized without specification of the range or final horizontal velocity component. Figures 5-9 show the results. Although the simulator trajectory climbed slightly steeper with a slight loss in range, the total time was the same. The theoretical optimum without constraint calls for a downward acceleration of about  $10g$  to bring the flight path angle to zero at the end of the climb. The simulation could obtain only about  $2g$  of downward acceleration causing an overshoot of the cruise altitude. The optimum violates several physical constraints. Not only does the aircraft go through the ground and sustain  $10g$  of acceleration, but the pitch angle becomes too large for passenger comfort and the velocity becomes large enough for compressibility effects to occur.

However, this method does get the aircraft to an altitude of 20,000 ft in only 53 sec. The resulting fuel consumption is equivalent to 87 miles of range. No other method was discovered with the simulator that required less fuel than this method. The broken lines in Figs. 5-9 show a case where the aircraft did not go below the ground. This nonoptimum example took 96 cruise miles of fuel.

Mehra and Bryson considered a vertical takeoff with a pitch angle of  $90^\circ$ . This allows the angle of attack and, in turn, the lift and induced drag to remain zero. Thus, the aircraft rose to 1000 ft with an essentially constant acceleration. After 16 sec the altitude of 1000 ft is reached with a vertical velocity of 125 fps. The optimum control theory solution started with the conditions at the end of 16 sec.

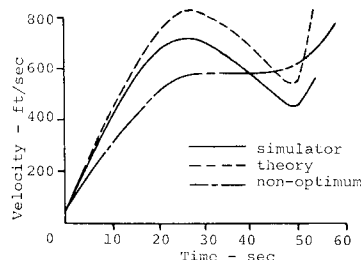
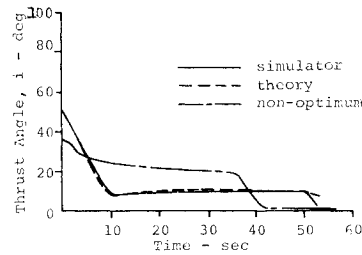


Fig. 7 Horizontal velocity for unconstrained takeoff.

**Fig. 8 Thrust angle for unconstrained takeoff.**



These initial conditions were also used for the simulator duplication. Again the simulator results were very close to the optimum control theory results as seen in Fig. 10. The same deviations as for the unconstrained takeoff were again present.

The aircraft climbs to the cruise altitude of 20,000 ft in 68 sec with this method, using 112 cruise miles of fuel. With these initial conditions, no better method of climbing was found with the simulator.

Further constraints were imposed on the takeoff to make it more physically realizable. The pitch angle was not allowed to exceed a comfortable level of  $\pm 30^\circ$ . The angle of attack was not allowed to exceed the maximum value of  $15^\circ$ . The velocity was kept below 600 fps. The aircraft was required to rise vertically to an altitude of 300 ft and then execute a  $20^\circ$  angle of climb to an altitude of 3000 ft. Horizontal flight had to be obtained at the end of the climb with a downward acceleration of  $1g$  or less.

This constrained takeoff was flown first with a constant thrust of  $1.25W$ . The resulting flight path and time histories for the flight variables are shown in Figs. 11–15. It took 91 sec or 151 cruise miles of fuel to reach 20,000 ft. This is 64 miles more than that required with the unconstrained optimum.

The case when the maximum thrust is  $1.10W$  was also considered. The flight was very similar to the one with a thrust of  $1.25W$  available. It took 114 sec or 165 cruise miles of fuel to reach 20,000 ft.

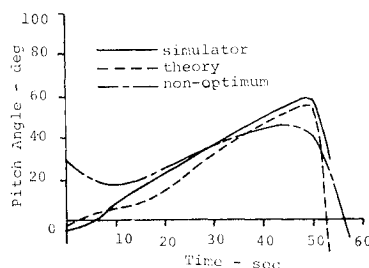
Climb angles other than  $20^\circ$  also were tried. The results are nearly the same for all climb angles less than  $30^\circ$  because of the velocity constraint, pitch angle constraint, and the acceleration constraint at the end of the climb. An  $8^\circ$  takeoff uses 5 cruise miles of fuel less than the  $20^\circ$  takeoff. Thus, a low climb angle requires somewhat less fuel.

Variable thrust was used next to try to reduce the fuel consumption. The results are shown by the dashed lines in Figs. 11–15. The thrust is kept at its maximum level of  $1.25W$  at first. Then it is beneficial to reduce the thrust when the maximum velocity is reached. In this manner, a saving of 36 miles over the similar constant thrust method resulted.

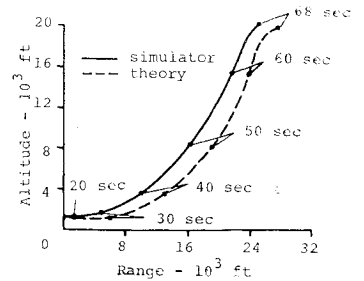
### Optimum Landing

The final touchdown of a VTOL vehicle is expected to be under visual conditions from a hover 20 to 40 ft above the touchdown point. The optimization problem is to get from cruise at 20,000 ft to hover at about 20 ft with minimum fuel. The unconstrained optimum would put the vehicle in a poweroff glide pulling up vertically from below the final touchdown point so as to reach zero velocity at the

**Fig. 9 Pitch angle for unconstrained takeoff.**

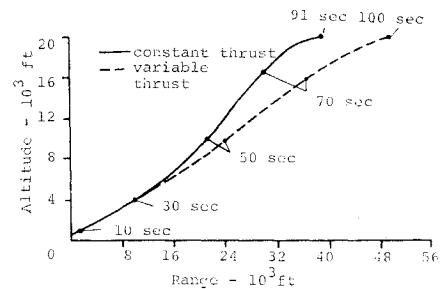


**Fig. 10 Flight path for vertical takeoff.**



final time without the expenditure of fuel. With the constraint that the vehicle remain above ground level, the poweroff glide needs to be terminated with thrust. The angle of attack should be maximum at the end of the poweroff glide, and the altitude should be as low as possible. This results in a minimum velocity to be nulled by thrusting. The maximum-range, poweroff glide is with an equilibrium flight path angle of  $-4.8^\circ$  for  $\alpha_{(L/D)_{max}}$ . At an altitude of 4000 ft, the glide is transitioned to  $-7.2^\circ$  with the angle of attack at its maximum value. The vertical component of velocity is  $-35$  fps at low altitude for both angles of attack.

The search for the optimum thrust maneuver was begun by keeping the thrust at its maximum level once it was applied. Various methods of employing the other two controls were used to determine the method that required

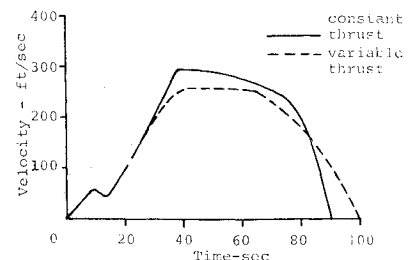


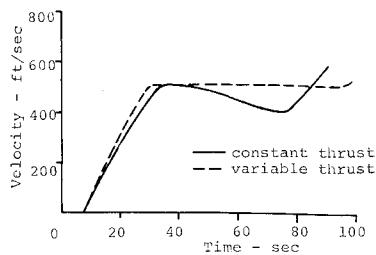
**Fig. 11 Flight path for constrained takeoff.**

the least fuel. The solid lines in Figs. 16 and 17 show the flight path and vertical velocity for the optimum landing method that was found. These curves begin at the end of the poweroff equilibrium glide. First, an oscillation is produced that allows the aircraft to level off. Twenty-two seconds later the thrust is applied at its maximum level of  $1.25W$  to bring the aircraft to a stop. The angle of attack is kept at its maximum value of  $15^\circ$  throughout, except when it is decreased to produce the oscillation. To produce the oscillation, the angle of attack is reduced to  $12^\circ$  and then brought back to  $15^\circ$  over a 10-sec interval. This disturbance occurs at an altitude of 1050 ft. The flight path angle decreases to a minimum value of  $-14^\circ$ . Then, when it has increased nearly to zero, thrust is applied. The thrust is applied at a low altitude of less than 20 ft and a range of 1500 ft from the landing point. In this manner the thrust has to be applied for only 7.9 sec. The fuel that is consumed is equivalent to 13 cruise miles.

The optimum method of control is the same if the maximum available thrust is less than  $1.25W$ . The dashed lines in

**Fig. 12 Vertical velocity for constrained takeoff.**



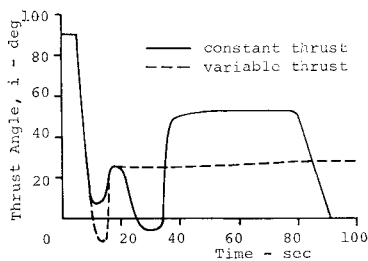


**Fig. 13 Horizontal velocity for constrained take-off.**

Figs. 16 and 17 show the case when the maximum thrust is only  $1.05 W$ . The thrust has to be applied for 15 sec. An additional 625 ft of range are required. The fuel needed for this case is equivalent to 17 cruise miles.

The broken lines in Figs. 16 and 17 show a landing in which the equilibrium glide is not disturbed before thrusting. The thrust is applied at the maximum level of  $1.25 W$ , the angle of attack is kept at its maximum value, and the flight path angle is kept constant at the equilibrium value. Thrusting is begun at an altitude of 335 ft and a range of 2700 ft from the landing point. Eleven seconds of thrust or 17 cruise miles of fuel are required.

Variable thrust was used to try to find a method of landing that requires less than 13 cruise miles of fuel. No such variable thrust method was found. It is reasonable that the

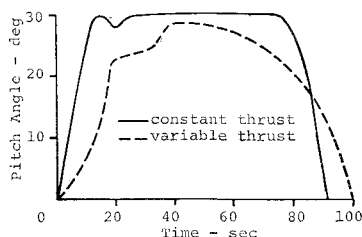


**Fig. 14 Thrust angle for constrained takeoff.**

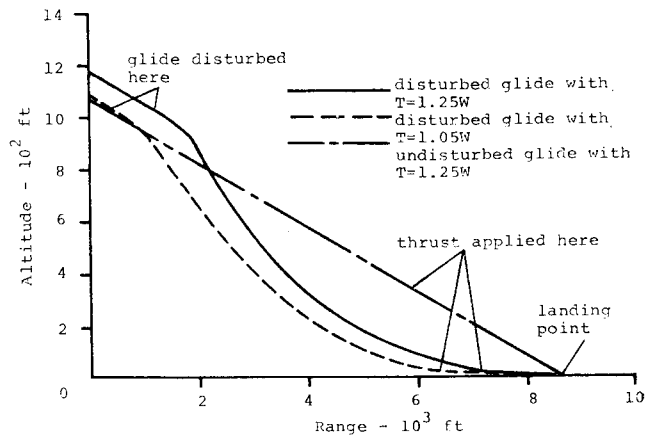
maximum thrust should be used in order to decrease the time that gravity has to be opposed.

The acceleration in the negative  $x$  direction produced by using constant maximum thrust is large for passenger comfort. An acceleration greater than  $0.7 g$  is produced with most of the maximum thrust methods. The optimum method produces an acceleration of  $1.20 g$  in the negative  $x$  direction.

The thrust was varied to keep this acceleration below  $0.5 g$ . It is brought rapidly to 25,000 lb. Then it was increased to the maximum of  $1.25 W$ , keeping the acceleration below  $0.5 g$ . The flight path angle was held constant and the angle of attack was kept at its maximum value. Both disturbed and undisturbed equilibrium glide landings were performed. In each case the fuel requirement was just slightly greater than that for constant maximum thrust. Thus, if the acceleration is constrained to be less than  $0.5 g$ , more time, altitude, and range are required; but the fuel penalty is not large. The variable thrust landings require only one cruise mile more fuel than the corresponding constant maximum thrust landing.



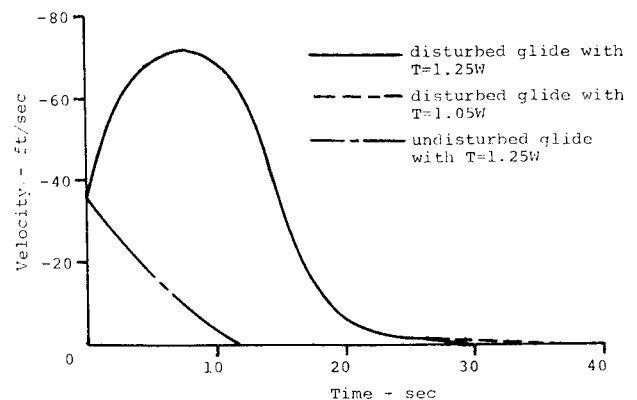
**Fig. 15 Pitch angle for constrained take-off.**



**Fig. 16 Flight path for constant thrust landing.**

### Steep Approaches

The aircraft was constrained to maintain constant flight path angles of  $-7^\circ$ ,  $-15^\circ$ , and  $-25^\circ$  below an altitude of 2000 ft. Each of these angles was flown with the pitch angle set to provide the maximum angle of attack of  $15^\circ$ . It was very difficult to keep the aircraft on the  $-15^\circ$  and  $-25^\circ$  flight paths by controlling the thrust magnitude and direction directly. These flight paths could be maintained only for several hundred feet of altitude before large oscillations were encountered. Other investigators<sup>7,8</sup> have reported the same difficulty in maintaining path control during steep approaches with several types of VTOL aircraft. A simple velocity control system was added in order to maintain the desired constant flight path angles. The velocity control system eliminated the difficulty in maintaining path control during steep approaches. Similar results have been reported by Rhodes and Tymczyszyn<sup>8</sup> by using a helicopter simulation. The advantages of velocity control have been known for some time, and systems have been developed for operation in military aircraft.<sup>9,10</sup> The diagram for the velocity control system is shown in Fig. 18. The error signals are obtained by comparing the desired velocity with the actual velocity. The vertical velocity component error signal produces a thrust direction input. The limiter keeps the thrust between zero and  $1.25 W$ . The angle of attack was maintained at its maximum value. This particular velocity control system will give economic operation at velocities below stall speed. A different control system would be called for at higher speeds. The  $-7^\circ$  approach required 19 cruise miles of fuel which was one cruise mile less efficient than direct control. The fuel cost increased with flight angle at an almost constant rate of 0.7 cruise mile per degree. Figure 19 shows the vertical profiles. The steeper approaches call for higher vertical velocities.



**Fig. 17 Vertical velocity for constant thrust landing.**

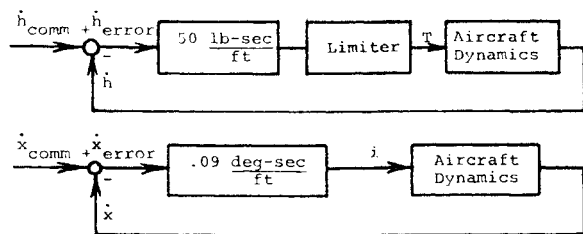


Fig. 18 Velocity control system.

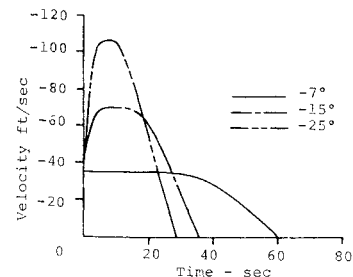
If the vertical velocity is constrained to be less than some small value below an initial altitude, then the aircraft will take the same length of time in performing the descent and the fuel cost will not be dependent upon approach angle. The fuel cost, however, is very large. A 20 fps descent from 1200 ft, for example, requires over 150 cruise miles of fuel. The steep approach angle and slow rate of descent force the aircraft velocity to be well below stall speed. Thrust must be approximately equal to weight, and the fuel cost goes to 79 cruise miles for each minute the aircraft spends in the descent. It can be seen that the fuel cost for the optimum landing is swamped by any constraint that requires below stall speed for longer than a single minute.

### Conclusions

To minimize fuel it is most important to minimize the time spent at airspeeds below conventional stall speed. Optimum flight below conventional stall speed requires that the angle of attack be held at its maximum limiting value. This is the most important consideration for optimization of the transition or for sustained flight at speeds just below conventional stall speed. Optimum methods of control during the takeoff and landing were found by trial and error. While an optimum found in this manner is not guaranteed to be the absolute mathematical optimum, the fuel costs under the various constraints are representative. Even modest constraints made large changes to the unconstrained fuel cost. Realistic constraints leave little room for fuel optimization.

Steep approaches increase the fuel cost about 0.7 cruise mile per degree for approaches greater than  $7^\circ$ . The vertical velocity also increases with steeper approaches. If the vertical velocity is constrained, the fuel cost increases greatly but loses its sensitivity to approach angle. Steep approaches require an automatic velocity control system to keep the aircraft on the prescribed flight path. The velocity control system makes flight path control relatively easy

Fig. 19 Vertical velocity for constant path angle landing.



when used with a position display showing altitude vs range. The use of a velocity system adds a negligible fuel cost.

### References

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